Introduction to Calculus and the Derivative

CHAPTER 5 OF "A MATHEMATICS COURSE FOR POLITICAL AND SOCIAL RESEARCH".

Definição formal da derivada

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(2)

a) $y = 6 \Rightarrow \lim_{h \to 0} \frac{6-6}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$,,

$f(x) = 6$

$f(x+h) = 6$

b) $f(x) = 3x^2 \Rightarrow \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h} = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$

$= \lim_{h \to 0} \frac{h(6x+3h)}{h} = \lim_{h \to 0} 6x + 3h = 6x$

c) $f(x) = x^3 - 2x^2 - 1 \Rightarrow \lim_{h \to 0} \frac{(x+h)^3 - 2(x+h)^2 - 1 - x^3 + 2x^2 + 1}{h}$

$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x^2 - 4xh - 2h^2 - x^3 + 2x^2}{h} =$

$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 4xh - 2h^2}{h} = \lim_{h \to 0} 3x^2 + 3xh + h^2 - 4x - 2h$

$= 3x^2 - 4x$

d) $f(x) = x^4 + 5x \Rightarrow$ agora já vamos fazer pelas regras

$\frac{d}{dx} f(x) = \frac{d}{dx} (x^4 + 5x) = 4x^3 + 5$

e) $f(x) = x^8 \Rightarrow \frac{d}{dx} (x^8) = 8x^7$

(4) Partial Derivatives

a) $f(x,z) = 3zx + 2z \Rightarrow \frac{\partial}{\partial x} (3zx + 2z) = 3z$

b) $f(x,z) = 9x^2 + 3z^2 \Rightarrow \frac{\partial}{\partial x} (f(x,z)) = 18x$

c) $f(x,z) = 5xz + 7xz^2 + 9x^2 \Rightarrow \frac{\partial}{\partial x} (f(x,z)) = 5z + 7z^2 + 9z x^{2-1}$